USN | | | | | | | | | | 10MAT11

First Semester B.E. Degree Examination, Dec.2015/Jan.2016
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting atleast TWO questions from each part.***
2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR sheet will not be valued.

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) If $y = \frac{1+x}{1-x}$, y_n = ______

A) $\frac{(-1)^{2n}n!}{(1-x)^{n+1}}$ B) $\frac{2(-1)^{2n}n!}{(1-x)^{n+1}}$ C) $\frac{2}{(1-x)^{n+1}}$ D) 0

ii) The condition between f(a) and f(b) in Lagrange's mean value theorem is ______

A) f(a) = f(b) B) f(a + b) = 0 C) f(a) ≠ f(b) D) f(a – b) = 0

iii) If $f(x) = e^x$, $g(x) = e^{-x}$ then Cauchy's mean value theorem the value of C in [a, b] is given by ______

A) $\frac{a}{2}$ B) $\frac{b}{2}$ C) $\frac{a-b}{2}$ D) $\frac{a+b}{2}$

iv) By Maclaurin's series the expression : $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\ldots$ is equal to____

A) e^x B) e^{-x} C) sin x D) cos x

b. If $y = \left[x + \sqrt{1+x^2}\right]^m$, prove that $(x^2+1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. **(06 Marks)**

c. Verify Rolle's theorem and hence find the number when $f(x) = e^x[\sin x - \cos x]$ defined in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$. **(06 Marks)**

d. Expand $\tan^{-1}x$ in the powers of x – 1 upto the term containing fourth degree. **(04 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\lim_{x\to 0} x^x$ is _____

A) –1 B) 0 C) 1 D) 2

ii) The angle between radius vector and the tangent of the curve $r = ae^{\theta \cot \alpha}$ is ______

A) 0 B) –α C) α D) $\frac{\alpha}{2}$

iii) The derivative of arc length of the curve $y^2 = 4ax$ is $\frac{ds}{dy}$ = ______

A) $\left[1+\frac{a}{x}\right]^{1/2}$ B) $\left[1+\frac{x}{a}\right]^{1/2}$ C) $\left[1-\frac{x}{a}\right]^{1/2}$ D) $\left[1-\frac{a}{x}\right]^{1/2}$

iv) Radius of curvature of the curve $y = x^2 - 3x + 1$ at (1, –1) is ______

A) $\sqrt{1/2}$ B) $1/2$ C) 2 D) $\sqrt{2}$.

b. Find the values of a and b when $\lim_{x\to 0}\left[\frac{x(1-a\cos x)+b\sin x}{x^3}\right]$ may be equal to $\frac{1}{3}$. **(06 Marks)**

c. Find the angle of intersection between two curves $r = a(1+\cos\theta)$ and $r = a(1-\cos\theta)$. **(06 Marks)**

d. Find the Radius of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. **(04 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) If $u = \log\left(\frac{x^2 + y^2}{xy}\right)$ then $\frac{\partial^2 u}{\partial y \partial x} =$ ______

A) $\frac{4xy}{(x^2 + y^2)^2}$ B) $\frac{-4xy}{(x^2 + y^2)^2}$ C) $\frac{4(x + y)}{(x^2 + y^2)^2}$ D) $\frac{4(x - y)}{(x^2 + y^2)^2}$

ii) If $x = r\cos\theta$, $y = r\sin\theta$, then the value of the Jacobian $J\left(\frac{x, y}{r, \theta}\right) =$ ______

A) $1/r$ B) $-2r$ C) $r/2$ D) r

iii) If $f(x, y) = x^3y^2 - x^4y^2 - x^3y^3$, one pair of stationary values are ______ = (x, y)

A) $\left(\frac{1}{2}, \frac{1}{3}\right)$ B) $\left(-\frac{1}{2}, -\frac{1}{3}\right)$ C) $\left(-\frac{1}{2}, \frac{1}{3}\right)$ D) $\left(\frac{1}{2}, -\frac{1}{3}\right)$

iv) If $x = 2$, $y = 1$, $\Delta x = \Delta y = 0.1$ be the error, then the error in A is ____ where A is the area of the rectangle

A) 0.1 B) 0.2 C) 0.3 D) 0.4.

b. If $u = x^2 + y^2 + z^2$ and $x = e^{2t}$, $y = e^{2t}\cos 2t$, $z = e^{2t}\sin 2t$, find $\frac{du}{dt}$ as the total derivative and verify by direct differentiation. **(06 Marks)**

c. If $U = x + 3y^2 - z^3$, $V = 4x^2yz$, $W = 2z^2 - xy$ evaluate $\frac{\partial(U, V, W)}{\partial(x, y, z)}$ at (1, −1, 0). **(06 Marks)**

d. The diameter and altitude of a can in the shape of a right circular cylinder are measured as 4 cms and 6 cms respectively. The possible error in each measurement is 0.1 cm. Find approximately the maximum possible error in the computed value of volume. **(04 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) If $\phi = x^2 + y^2 + z^2$ then $|\nabla\phi|$ at (1, 1, 1) is ______

A) 8 B) 10 C) 12 D) 14

ii) If $\vec{u} = (x + 3y)i + (y - 2z)j + (x + \lambda z)k$ is said to be solenoidal then the value of $\lambda =$ ______

A) −1 B) −2 C) −3 D) −4

iii) Curl (grad ϕ) = ______

A) 1 B) −1 C) $1/2$ D) 0

iv) In orthogonal curvilinear coordinates if the Cartesian coordinates x, y, z are functions of u, v, w then the value of the Jacobian $J = \frac{\partial(x, y, z)}{\partial(u, v, w)}$ is ______

A) $= 0$ B) $\neq 0$ C) < 0 D) > 0.

b. Find the constants a, b, c such that the vector

$\vec{F} = (x + y + az)i + (x + cy + 2z)k + (bx + 2y - z)j$ is irrotational. **(06 Marks)**

c. Prove that $\nabla \cdot \left(\nabla \times \vec{A}\right) = 0$ with $A = A_1 i + A_2 j + A_3 k$. **(06 Marks)**

d. Prove that a spherical coordinate system is orthogonal. **(04 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) If $F(a)=\int_0^1 \frac{x^a-1}{\log x}dx$ then $F'(a)=$ ______

A) $\frac{1}{\alpha-1}$ B) $\frac{1}{\alpha+1}$ C) $\frac{\alpha}{\alpha-1}$ D) $\frac{1}{\alpha}$

ii) The value of $\int_0^{\pi/2} \sin^6\theta d\theta=$ ______

A) $\frac{5}{16}$ B) $\frac{5}{14}$ C) $\frac{5}{13}$ D) $\frac{5}{11}$

iii) The curve $x^3+y^3=3axy$ is symmetric about

A) $x+y=a$ B) $x-y=a$ C) $y=x$ D) $y=x^2$

iv) The equation to find the area of $r=a(1+\cos\theta)$ is

A) $\int_0^{\pi} r dr$ B) $\int_0^{\pi} r^2 dr$ C) $\int_0^{\pi} 2r^2 d\theta$ D) $\int_0^{\pi} r^2 d\theta$.

b. Evaluate : $\int_0^{\infty} \frac{\tan^{-1}(ax)}{x(1+x^2)}dx$ using the method of differentiation under integral sign. **(06 Marks)**

c. Evaluate $\int_0^{2a} x^2\sqrt{2ax-x^2}dx$. **(06 Marks)**

d. Find the area of the surface generated by revolving one arch of the cycloid $x=a(1-\sin t)$, $y=a(1-\cos t)$ in $0\le t\le 2\pi$. **(04 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) The solution of $\frac{x^2}{y}dx+\frac{y^2}{x}dy=0$ is = ______

A) $x^4+y^4=x$ B) $x^4+y^4=y$ C) $x^4+y^4=0$ D) $\frac{x^4}{4}+\frac{y^4}{4}=c$

ii) The differential equation $(1+xy^2)dx+(1+x^2y)dy=0$ is said to be ______

A) Homogeneous B) Linear C) Exact D) Legendre's

iii) The integrating factor of $(x+2y^3)\frac{dy}{dx}=y$ is ______

A) $\frac{1}{y}$ B) y C) $\frac{1}{y^2}$ D) y^2

iv) Orthogonal trajectory of $r=a\theta$ is given as $re^{\theta^2/2}=$ ______

A) e^2 B) e^c C) e^3 D) 0

b. Solve $(x-y+1)dx-(x+2y-3)dy=0$. **(06 Marks)**

c. Solve $(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$. **(06 Marks)**

d. Find the orthogonal trajectory of the curve $r^n=a^n\cos n\theta$. **(04 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) Normal form of the matrix is denoted by

A) $\begin{bmatrix} I_r & 0 \\ 0 & I_r \end{bmatrix}$ B) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ D) $\begin{bmatrix} I_r & I_r \\ 0 & 0 \end{bmatrix}$

ii) The rank of the matrix is ______ when $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

A) 3 B) 2 C) 1 D) 0

iii) If the system of linear equations is said to be inconsistent then Ranks of the given matrix and augmented matrix are

A) same B) not equal C) trivial D) non trivial

iv) In Gauss Jordon method, the given square matrix is reduced to ______ form.

A) Row matrix B) Column matrix C) Null matrix D) Diagonal matrix.

b. Find the rank of the matrix $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$. **(06 Marks)**

c. Test for consistency and solve :

$5x + 3y + 7z = 4$

$3x + 26y + 2z = 9$

$7x + 2y + 10z = 5$. **(06 Marks)**

d. Solve the system of linear equations by Gauss – Jordan method given that :

$2x + 5y + 7z = 52$

$2x + y - z = 0$

$x + y + z = 9$. **(04 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) Eigen values of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ are

A) 6, –1 B) 6, 1 C) 6, 0 D) –6, 0

ii) A homogeneous expression of the second degree in any number of variables is called

A) spectral form B) diagonal form C) symmetric form D) quadratic form

iii) If $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ is the quadratic form of the matrix A then the eigen values are ______

A) 3, 5, 3 B) –1, 1, –1 C) 3, –1, 1 D) –2, 2, –2

iv) The matrix P which diagonalises the square matrix A is called the ______ matrix.

A) singular B) model matrix of A C) unit matrix D) power of a matrix.

b. Find the eigen values and eigen vectors of the matrix : $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(06 Marks)**

c. Reduce $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ to the diagonal form. **(06 Marks)**

d. Reduce $3x^2 + 3z^2 + 4xy + 8xy + 8yz$ into canonical form. **(04 Marks)**

* * * * *